(Pages : 4) **T – 1607**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course X

MM 1642 : COMPLEX ANALYSIS – II

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions.

1. State Morera's theorem.

2. State generalized Cauchy's integral formula.

3. Evaluate $\int_{|z|=4} \frac{1}{z-2} dz$.

4. Define uniform convergence in sequence.

5. Find $\sum_{j=0}^{\infty} \left(\frac{1}{3}\right)^j$.

P.T.O.

6. Using the ratio test, show that $\sum_{j=0}^{\infty} \frac{j^2}{4^j}$ converges

7. Find the Maclaurin's series for sinz.

8. Find the singularities of $f(z) = \frac{\cos z}{z^2 (z-\pi)^3}$.

9. Define pole. Give an example.

10. Find the poles of $f(z) = \frac{z^2}{z^2 + 4}$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions.

11. Compute $\int_{|z|=1} \frac{e^{5z}}{z^3} dz$.

12. Show that $\int_{|z|=3} \frac{e^z}{z-2} dz = 2\pi i e^2$.

13. Find $\int_C \frac{dz}{z-1}$ where C is the circle $|z| = 3$.

14. Show that $1 - c - c^2 + ... = \frac{1}{1-c}$, if $|c| < 1$.

15. If $\sum_{j=0}^{\infty} c_j$ sums to S and λ is any complex number then show that $\sum_{j=0}^{\infty} \lambda c_j$ sums to λS.

16. Prove that $\lim_{n \to \infty} (n!)^{\frac{1}{n}} = \infty$.

17. Expand $e^{\frac{1}{z}}$ in a Laurent series around z = 0.

18. Find the residue of *f*(z) tanz at $z = \frac{\pi}{2}$.

19. Find the residue at $z = 0$ of $f(z) = \frac{5z-2}{z(z-1)}$.

20. Determine the order of each pole and the value of residue there for $f(z) = \frac{1-e^{2z}}{z^4}$.

21. Prove that $\lim_{n \to \infty} (n!)^{\frac{1}{n}} = \infty$.

22. Find the Maclaurin series expansion of sinhz.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions.

23. Find $\int_C \frac{e^z + \sin z}{z} dz$ where *C* is the circle $|z-2| = 3$.

24. If *f* is analytic in a domain D, show that all its derivatives *f*′, *f*″..... exist and are analytic in D.

25. Evaluate $\int_{|z|=3} \frac{z^2+5}{(z-2)^2} dz$.

26. State and prove ratio test.

27. Find the first five terms of the Maclaurin's series for tanz.

28. If R is the radius of convergence of $\sum a_n z^n$ then what is the redii of convergence of $\sum a_n^2 z_n$ and $\sum a_n z^{2n}$.

29. Compute the residue at singularity of $f(z) = \dfrac{\cos z}{z^2(z-\pi)^3}$.

30. Find PV $\int_{-\infty}^{\infty} \dfrac{x \sin x}{(1+x^2)} dx$.

31. Evaluate $\int_{|z-1|=1} \dfrac{2z^2+z}{z^2+1} dz$ using Cauchy Residue theorem.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions.

32. State and prove Cauchy's integral formula.

33. (a) State Picard's theorem and verify it for $e^{\frac{1}{z}}$ near $z = 0$.

(b) Explain zeroes and different types of singularities with examples.

34. (a) State and prove Cauchy Residue theorem.

(b) Using Cauchy Residue theorem, evaluate $\oint_{|z|=2} \dfrac{1-2z}{z(z-1)(z-3)} dz$.

35. Evaluate $\int_0^{\pi} \dfrac{d\theta}{2-\cos\theta}$.

(2 × 15 = 30 Marks)

(Pages : 4) **T – 1608**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

MM 1643 : ABSTRACT ALGEBRA : RING THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the first ten questions. Each carries **1** mark.

1. Give a proper nontrivial subring of Z_8.

2. Give an example for an integral domain.

3. Let A be a subring of ring R. If $r \in R$, $a \in A$ implies $ra \in A$ then A is called __________.

4. Give an example for a ring R and subring of it that is not an ideal.

5. Define ring homomorphism.

6. Which is true: Z homomorphic to Z_n or Z isomorphic to Z_n?

7. State the division algorithm for $F[x]$ where F is a field.

8. Define associates in an integral domain.

P.T.O.

9. Define Euclidean Domain.

10. In what type of integral domain, irreducibles and primes are the same?

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each carries **2** marks.

11. Show that in a ring R, if $a, b \in R$, then $a.0 = 0$ and $a(-b) = (-ab)$.

12. Show that left (multiplicative) cancellation holds in an integral domain.

13. What is meant by an ideal generated by $a_1, a_2, ...a_n$ in a ring R? Find ideal generated by x^2 in $Z[x]$.

14. If A and B are ideals in a ring R, show that $A + B$ is an ideal.

15. If R is a commutative ring with characteristic 2, show that $a \to a^2$ is a homomorphism on R.

16. If rings R, S are isomorphic, show that $R[x]$ and $S[x]$ are isomorphic.

17. Is $a + ib \to |a + ib|$ a homomorphism from the set of all complex numbers C to C? Justify.

18. Is $x^2 + 1$ irreducible over Z_3? Justify.

19. Define a unique factorization domain. Give an example.

20. Show that if F is a field, then $F[x]$ is a Euclidean domain.

21. On an integral domain D, define $a \sim b$ if a and b are associates. Show that this is an equivalence relation on D.

22. Give two factorizations of 21 in $Z\left[\sqrt{-5}\right]$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each carries **4** marks.

23. Show that every nonzero element of Z_n is either a unit or a zero divisor. Is this true for Z?

24. Show that if R is a ring with unity, then R/A is an integral domain if and only if A is a prime ideal.

25. Let $R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in Z \right\}$ and let I be a subset of R with even entries at all the places in its elements. Show that it is an ideal of R. Find out the elements in $\frac{R}{I}$.

26. Show that $f(x) = 5x$ is a ring homomorphism from Z_4 to Z_{10}.

27. If D is an integral domain, prove that D [x] is also an integral domain.

28. Show that the product of two primitive polynomials is primitive.

29. Show that every Euclidean domain is a PID.

30. In a PID, show that every strictly increasing chain of ideals must be finite in length.

31. Show that the integral domain $Z\left[\sqrt{-5}\right]$ is not a UFD.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

32. (a) If R is a commutative ring with unity, A an ideal in it, show that $\frac{R}{A}$ is a field if and only if A is maximal.

    (b) Show that $< x >$ is a prime ideal in $Z[x]$, but not maximal.

33. (a) Describe the subrings of the ring of all integers.

(b) Which of these a prime ideals? Why?

(c) Let $a \in R$, a ring. Is $S = \{x \in R : ax = 0\}$ a subring? Is this an ideal? Justify your answers.

34. State and prove the theorem on unique factorization of nonzero, non unit elements in $Z[x]$.

35. Prove that every PID is a UFD.

(2 × 15 = 30 Marks)

(Pages : 6) **T – 1610**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course XII

MM 1644 : LINEAR ALGEBRA

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. Each question carries **1** mark.

1. Find the intersection of the lines $x + y = 3$ and $x - y = 1$.

2. For which values of α, is there a whole line of solution for the following equation

$$\alpha x + 4y = 0$$
$$4x + 4\alpha y = 0$$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ then express AB as the linear combination of columns of A.

4. Define Subspace of a vector space over a field F.

P.T.O.

5. Find a basis for the vector space $\mathbb{R}^3$ over $\mathbb{R}$.

6. Find the null space of the matrix $A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$.

7. Define the rank of a matrix.

8. Find the determinant of the matrix $\begin{bmatrix} -1 & 2 & 1 \\ 2 & -4 & -2 \\ 1 & 1 & 7 \end{bmatrix}$.

9. Find the Eigen values of $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

10. What is the relation between trace of a matrix and its Eigen values?

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. Find the values of β such that the system,

$2x - 6y = \beta$

$-x + 3y = -4$

(a) Has no solution (b) Has infinite solution

12. Find two points on the line of intersection of the three planes $t = 0$ and $z = 0$ and $x + y + z + t = 1$ in four-dimensional space.

13. What are the elementary transformation in order to make the system

$x + 2y = 3$

$3x + 5y = 8$, an upper triangular system. Also write the transformed system and find the solution.

14. Is matrix multiplication commutative? Justify.

15. Let $v_1, v_2, \ldots v_n$ are linearly independent vectors in a vector space V. Then, what is the dimension of the span of these vectors? If we add a vector v_{n+1}, where v_{n+1} is a linear combination of $v_1, v_2, \ldots \ldots, v_n$, then what the change in the dimension of the span of these $(n+1)$ vectors?

16. Write down the 2 by 2 matrices A and B that have entries $a_{ij} = i + j$ and $b_{ij} = (-1)^{i+j}$ multiply them to find AB and $|AB|$.

17. Check whether the set $W = \{(x, y) \in \mathbb{R}^2 ; x + y = 1\}$ is a subspace of $\mathbb{R}^2$ or not?

18. Show that the columns of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ are linearly independent.

19. Using Crammers Rule solve $x_1 + 3x_2 = 0,\ 2x_1 + 4x_2 = 6$.

20. Draw the triangle with vertices $A = (0,0), B = (2,2)$, and $C = (-1,3)$. Find its area.

21. Find the Eigen values of the matrix A and A^2 if $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

22. Give examples of A and B such that, $A + B$ is not invertible although A and B are invertible.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Apply Elimination and back substitution to solve

$$x - 2y + z = 0;\ \ 2y - 8z = 8;\ \ -4x + 5y + 9z = -9$$

24. Prove that the eigenvalues of a triangular matrix are the entries on its main diagonal.

25. Find L and U such that $LU = A$; where $A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$.

26. The matrix that rotates the $x - y$ plane by an angle θ is $A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

(a) Verify $A(\theta_1)A(\theta_2) = A(\theta_1 + \theta_2)$

(b) Find $A(\theta)A(-\theta)$

27. Find the reduced echelon form of $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$. What is the rank?

28. Find the dimension and basis of subspaces

(a) column space and

(b) row space – of the matrix $A = \begin{bmatrix} 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

29. Find the Eigen values of matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

30. Let $u = [1, -2, -5]^T$, $v = [2, 5, 6]^T$ and $w = [7, 4, -3]^T$. Verify whether $\{u, v, w\}$ is independent or not if it is not independent, express w as the linear combination of u and v.

31. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map such that $T(1, 0) = (2, 3), T(0, 1) = (1, 4)$. Find T.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) Under what condition on b_1, b_2, b_3 is the following system solvable? Include b as a fourth column in $[A\,b]$. Find all solutions when that condition holds :

$$\begin{aligned} x + 2y - 2z &= b_1 \\ 2x + 5y - 4z &= b_2 \\ 4x + 9y - 8z &= b_3 \end{aligned}$$

(b) What conditions on b_1, b_2, b_3, b_4 make the following system solvable? Find x in that case.

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 2 & 5 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

33. Diagonalize the following matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

34. Reduce A to U and find det A = product of the pivots if

(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

35. (a) Show that $S = \{(1,1,1),(1,1,0),(1,0,0)\}$ form a basis for $\mathbb{R}^3$.

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$

(i) Show that T is Linear

(ii) Find the null space of T (Kernel of T)

(2 × 15 = 30 Marks)

(Pages : 4) **T – 1611**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course XIII

MM 1645 : INTEGRAL TRANSFORMS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find $\mathcal{L}\{te^{2t}\}$.

2. Find $\mathcal{L}\{\sin t \cos t\}$.

3. If $\mathcal{L}\{f(t)\} = F(s)$, then find $\mathcal{L}\{tf(t)\}$.

4. Write the second shifting property of Laplace transform.

5. Find $\mathcal{L}^{-1}\left\{\dfrac{2+s}{s^2+1}\right\}$.

6. Find the period of $f(x) = \cos 2x$.

7. Write an example for even function.

8. Write the Fourier series expansion of an even periodic function with period 2π.

9. Write the Fourier sine transform of e^{-ax}, $a > 0$.

10. Define Fourier transform of a function $f(x)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These question carries **2** marks each.

11. Using Definition, find Laplace transform of $f(t) = t$.

12. Find $\mathcal{L}\{\sinh t + 3\cos 2t + te^{t}\}$.

13. State property : Laplace Transform of derivative of a function. Hence write the Laplace transform of second derivative $F''(t)$ of $f(t)$.

14. Find $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s-1)}\right\}$.

15. Using the concept of Laplace transform find $\int_{0^+}^{\infty} e^{-2t} t\cos t \, dt$.

16. Using the Laplace transform of integral, find $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+w^2)}\right\}$.

17. Write second shifting theorem of Laplace transform.

18. Find Fourier series of the function $f(x) = x; -\pi < x < \pi$.

19. Write the Fourier series expansion of a 2T periodic function defined in $(-T, T)$.

20. Find Fourier cosine series of $f(x) = e^{2x}; 0 < x < 1$.

21. Find Fourier cosine transform of $f(x)=\begin{cases} k\ ; 0<x<a \\ 0\ ; \quad x>a \end{cases}$

22. Show that $\mathcal{F}_C\{f'(x)\} = w\,\mathcal{F}_S\{f(x)\} - \sqrt{\frac{2}{\pi}}\,f(0)$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Define Dirac's Delta function and find its Laplace transform.

24. State and prove first shifting theorem of Laplace Transform.

25. Using Laplace transform, solve the differential equation $y'' + 4y = 4t$, $y(0)=1$ and $y'(0)=5$.

26. Using Laplace transform solve the integral equation $y(x) = x^3 + \int_0^x \sin(x-t)\,y(t)\,dt$.

27. Evaluate $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$, using convolution property.

28. Represent function $f(x)=\begin{cases} -1 \ \text{for}\ -\pi<x<0 \\ 0 \ \ \text{for}\ x=0 \\ 1 \ \ \text{for}\ 0<x<\pi \end{cases}$ as a Fourier series.

29. Find Fourier integral representation of function of $f(x)=\begin{cases} 1 \ \text{if}\ |x|<1 \\ 0 \ \text{if}\ |x|>1 \end{cases}$

30. Find the Fourier Transform of $f(x)=1$ if $|x|<1$ and $f(x)=0$ otherwise.

31. Show that : $\mathcal{F}\{f'(x)\} = iw\,\mathcal{F}\{f(x)\}$, where $f(x)\to 0$ as $|x|\to\infty$ and $f'(x)$ is absolutely integrable over x.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These question carries **15** marks each.

32. Find

(a) $\mathcal{L}\{f(t)\}$ if $f(t)=\begin{cases}1, 0<t<\pi \\ 0, \pi<t<2\pi \\ \sin t, t>2\pi\end{cases}$

(b) $\mathcal{L}\left\{\frac{\sin t}{t}\right\}$

(c) $\mathcal{L}^{-1}\left\{\ln\frac{s+a}{s+b}\right\}$

33. (a) Deduce a formula to calculate the Laplace transform of the n^{th} derivative $f^{n}(t)$ of a function $f(t)$.

(b) Using Laplace transform solve the system of differential equations

$y_1' + y_2 = 1$ and $y_2' - y_1 + 4e^{t} = 0$ given $y_1(0)=0$, $y_2(0)=0$.

34. Find half range Fourier sine and cosine series of $f(x)=\begin{cases}\frac{2k}{L}x & \text{if } 0<x<\frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2}<x<L\end{cases}$

35. Using Fourier integral representation, show that $\int_0^{\infty}\frac{\cos\frac{\pi\omega}{2}}{1-\omega^2}\cos\omega x \, d\omega=\begin{cases}\frac{\pi}{2}\cos x; & |x|\le\frac{\pi}{2} \\ 0 & ; |x|>\frac{\pi}{2}\end{cases}$.

(2 × 15 = 30 Marks)

(Pages : 4) **T – 1612**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Elective Course

MM 1661.1 : GRAPH THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory

They carry **1** mark each.

1. What is an isolated vertex?
2. Define a complete graph.
3. State first theorem of graph theory.
4. Define trail.
5. Draw a tree with five vertices.
6. A graph G is called Hamiltonian if ————
7. State travelling salesman problem.
8. State Euler's formula for plane graphs.
9. Define degree of a face of a plane graph.
10. Give an example of an Eulerian graph.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer any **eight** questions

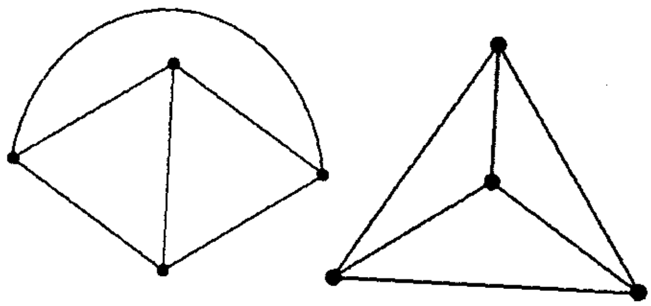
These questions carry **2** marks each.

11. Draw K_4.

12. Prove that in a tree, there is precisely one path between two distinct vertices.

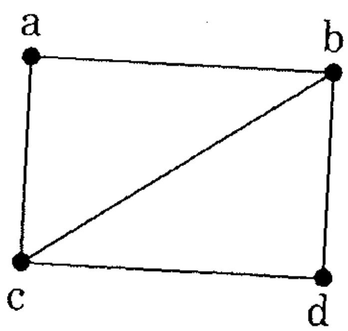
13. Prove that if G is a tree with n vertices, then G is an acyclic graph with $(n-1)$ edges.

14. Show that the following two graphs ate isomorphic:



15. Define k-regular graph and draw a 2-regular graph.

16. Show that in the following graph, sum of degrees of vertices is even.



17. Prove that if a connected graph G is Euler, then the degree of every vertex is even.

18. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.

19. Prove that redrawings of the same planar graph have same number of faces.

20. Explain travelling salesman problem.

21. Draw a planar graph and show that it's subgraphs are also planar.

22. State Kuratowski's theorem.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions

These questions carry **4** marks each.

23. Define graph isomorphism and give two isomorphic graphs with four vertices.

24. What is a spanning subgraph? Draw a spanning subgraph of K_4.

25. Prove that any tree with at least two vertices has more than one vertex of degree one.

26. Prove that a connected graph is a tree if and only if every edge is a bridge.

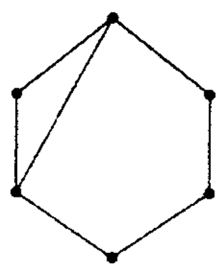
27. Prove that if a simple graph with at least three vertices is 2-connected if for each pair of distinct vertices *u* and *v* of *G*, there are two internally disjoint *u*-*v* paths in *G*.

28. Explain Chinese Postman problem.

29. Draw a Hamiltonian graph with six vertices.

30. Show that $K_{3,3}$ is non-planar.

31. Draw the closure of the following graph:



(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions

These questions carry **15** marks each.

32. Prove that a graph G is connected if and only if it has a spanning tree.

33. Prove that a tree with n vertices has precisely $(n\text{-}1)$ edges.

34. Prove that a connected graph G is Euler if and only if degree of every vertex of G is even.

35. Prove that if G is a connected plane graph and let n, e and f denote the number of vertices, edges and faces of G respectively, then $n\text{-}e + f\text{=}2$.

(2 × 15 = 30 Marks)

(Pages : 4) **T – 1605**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS – II

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. State composition of continuous functions theorem.

2. State extreme value theorem for continuous functions.

3. Define uniform continuity and give an example.

4. State intermediate value theorem.

5. Find the 10^{th} derivative of $f(x) = x^5 + 4x^2 + 1$.

6. Give an example of a monotone function.

7. When we say that a function is Riemann integrable.

8. Give an example of a set of measure 0.

P.T.O.

9. If $\int_a^b f = 10$, then $\int_b^a f = \cdots$.

10. State Lebesgue's Theorem.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. State sequential criterion for functional limits.

12. Evaluate $\lim_{x \to \pi} (x + \sin x)$.

13. Construct two functions f and g, neither of which is continuous at 0 but $f(x) + g(x)$ is continuous at 0.

14. Whether there exists a continuous function defined on a closed interval with range equal to {1,2,3}.

15. Define Lipschitz Function and give an example of a function which is uniformly continuous but not Lipschitz.

16. Define removable discontinuity with an example.

17. State Darboux's Theorem.

18. State Mean Value Theorem.

19. Find $\lim_{x \to 1} \left(\frac{1-x}{\ln x} \right)$.

20. If P_1 and P_2 are any two partitions of [a, b], then prove that $L(f, P_1) \leq U(f, P_2)$.

21. Distinguish between upper integral and lower integral.

22. If $\int_1^4 f = 4$ and $\int_2^4 f = 1$, then find $\int_1^2 f$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Using $\varepsilon - \delta$ definition prove that $\lim_{x \to 2}(3x+4) = 10$.

24. Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.

25. Is the converse of intermediate value theorem true? Justify your claim.

26. Let *f* be differentiable on an open interval *(a, b)*. If *f* attains a maximum value at some point $c \in (a,b)$ then prove that $f'(c) = 0$.

27. State and prove Rolle's theorem.

28. If $f : A \to R$ is differentiable at a point $c \in A$. then prove that *f* is continuous at *c*. Is the converse true? Justify your answer.

29. If $g : A \to R$ is differentiable on an interval *A* and satisfies $g'(x) = 0$ for all $x \in A$, then prove that $g(x) = k$ for some constant $k \in R$.

30. Assume that $f_n \to f$ uniformly on [*a*, *b*] and that each f_n is integrable. Prove that *f* is integrable and $\lim_{n \to \infty} \int_a^b f_n = \int_a^b f$.

31. Prove that the Dirichlet's function $g(x) = \begin{cases} 1 \text{ for } x \text{ rational} \\ 0 \text{ for } x \text{ irrational} \end{cases}$ is not integrable.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. Let $f: A \to R$ be continuous on *A*. If $K \subseteq A$ is compact, then prove that *f(K)* is also compact.

33. State and prove chain rule for derivatives.

34. (a) Prove that a bounded function *f* is integrable on [*a*, *b*] if and only if, for every $\varepsilon > 0$, there exists a partition P_ε of [*a*, *b*] such that $U(f, P_\varepsilon) - L(f, P_\varepsilon) < \varepsilon$.

    (b) Prove that if *f* is continuous on [*a*, *b*], then it is integrable.

35. State and prove the fundamental theorem of integral calculus.

(2 × 15 = 30 Marks)
